

Let's discuss the idea of "range" today. It is simply the difference between the smallest and the greatest number in a set. Consider the following examples:

Range of {2, 6, 10, 25, 50} is $50 - 2 = 48$

Range of {-20, 100, 80, 30, 600} is $600 - (-20) = 620$

and so on...

That's all the theory we have on the concept of range! So let's jump on to some questions now (therein lies the challenge)!

Question 1: Which of the following cannot be the range of a set consisting of 5 odd multiples of 9?

- (A) 72
- (B) 144
- (C) 288
- (D) 324
- (E) 436

Solution:

There are infinite possibilities regarding the multiples of 9 that can be included in the set. The set could be any one of the following (or any one of the other infinite possibilities):

$S = \{9, 27, 45, 63, 81\}$ or

$S = \{9, 63, 81, 99, 153\}$ or

$S = \{99, 135, 153, 243, 1071\}$

The range in each case will be different. The question asks us for the option that 'cannot' be the range. Let's figure out the constraints on the range.

A set consisting of only odd multiples of 9 will have a range that is an even number (Odd Number – Odd Number = Even number)

Also, the range will be a multiple of 9 since both, the smallest and the greatest numbers, will be multiples of 9. So their difference will also be a multiple of 9.

Only one option will not satisfy these constraints. Do you remember the divisibility rule of 9? The sum of the digits of the number should be divisible by 9 for the number to be divisible by 9. The sum of the digits of 436 is $4 + 3 + 6 = 13$ which is not divisible by 9. Hence 436 cannot be divisible by 9 and therefore, cannot be the range of the set.

Answer (E).

On to another one now:

Question 2: If the arithmetic mean of n consecutive odd integers is 20, what is the greatest of the integers?

- (1) The range of the n integers is 18.

(2) The least of the n integers is 11.

Solution: We have discussed mean in case of arithmetic progressions in the previous posts. If mean of consecutive odd integers is 20, what do you think the integers will look like?

19, 21 or

17, 19, 21, 23 or

15, 17, 19, 21, 23, 25 or

13, 15, 17, 19, 21, 23, 25, 27 or

11, 13, 15, 17, 19, 21, 23, 25, 27, 29

etc.

Does it make sense that the required numbers will represent one such sequence? The numbers in the sequence will be equally distributed around 20. Every time you add a number to the left, you need to add one to the right to keep the mean 20. The smallest sequence will have 2 numbers 19 and 21, the largest will have infinite numbers. Did you notice that each one of these sequences has a unique “range,” a unique “least number” and a unique “greatest number?” So if you are given any one statistic of the sequence, you will know the entire desired sequence.

Statement 1: Only one possible sequence: 11, 13, 15, 17, 19, 21, 23, 25, 27, 29 will have the range 18. The greatest number here is 29. This statement alone is sufficient.

Statement 2: Only one possible sequence: 11, 13, 15, 17, 19, 21, 23, 25, 27, 29 will have 11 as the least number. The greatest number here is 29. This statement alone is sufficient too.

Answer (D).

Note that you don’t actually have to find the exact sequence. All you need to understand is that each sequence will have a unique “range” and a unique “least number.”

That’s all for this week. It wasn’t very dense but we will more than make up for it next week (that’s when we start with standard deviation)! Keep practicing!